Star Related Analytic Mean Square-Cordial Graphs

Dr. A. Nellai Murugan
Department of Mathematics, V.O.Chidambaram College, Tuticorin 628008, Tamilnadu, India.

N. Roselin
Department of Mathematics, V.O.Chidambaram College, Tuticorin 628008, India.

Abstract – Let $G = (V, E)$ be a graph with p vertices and q edges. An Analytic Mean Square-Cordial Labeling of a Graph G with vertex set V is a bijection from V to $\{0, 1\}$ such that each edge uv is assigned the label $f(uv) = |\lceil f(u)^2 - f(v)^2/2 \rceil|$ with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differby atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differby atmost 1. The graph that admits an Analytic Mean Square-Cordial Labeling is called Analytic Mean Square-Cordial Graph. In this paper, we proved that Star related graphs Star$K_{1,n}$, Subdivided Star$< K_{1,n}: n >$, Tree $Tr(n)$ are Analytic Mean Square-Cordial Graphs.

Index Terms – Star, Subdivided Star, Tree, Analytic Mean Square-Cordial Graph, Analytic Mean Square-Cordial Labeling.

2000 Mathematics Subject classification 05C78.

1. INTRODUCTION

A Graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair $e = \{uv\}$ of vertices in E is called edges or a line of G. In this paper, we proved that Star related graphsStar $K_{1,n}$, Subdivided Star$< K_{1,n}: n >$, Tree $Tr(n)$are Analytic Mean Square-Cordial Graphs. For graph theory terminology, we follow [2].

2. PRELIMINARIES

Let $G = (V, E)$ be a graph with p vertices and q edges. An Analytic Mean Square-Cordial Labeling of a Graph G with vertex set V is a bijection from V to $\{0,1\}$ such that each edge uv is assigned the label $f(uv) = |\lceil f(u)^2 - f(v)^2/2 \rceil|$ with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differby atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differby atmost 1.

The graph that admits an Analytic Mean Square-Cordial Labeling is called Analytic Mean Square-Cordial Graph. In this paper, we proved that Star related graphsStar $K_{1,n}$, Subdivided Star$< K_{1,n}: n >$, Tree $Tr(n)$are Analytic Mean Square-Cordial Graphs.

Definition: 2.1

A bipartite graph is a graph whose vertex set $V(G)$ can be partitioned into two subsetsV_1 and V_2 such that every edge of G has one end in V_1 and the other end inV_2 ; (V_1, V_2) is called a bipartition of G. If further,every vertex of V_1 is joined to all the vertices of V_2, then Gis called a complete bipartite graph. The complete bipartite graph with bipartition(V_1,V_2) such that $|V_1|$ = m and $|V_2|$ = n is denoted by $K_{m,n}$. A complete bipartite graph $K_{1,n}$ or $K_{n,1}$ (or) S_n is called a star.s

Definition:2.2

Subdivided star is a graph obtained as one point union of n paths of path length 2. It is denoted by $< K_{1,n}: n >$

Definition:2.3

Let Trbe any tree. Denote the tree obtained fromTr by considering two copies of $Tr$by adding an edge between them by $Tr(2)$and in general the graph obtained from $Tr(n-1)$and$Tr$by adding an edge between them is denoted by $Tr(n)$.

3. MAIN RESULTS

Theorem: 3.1

Star $K_{1,n}(odd)$is Analytic Mean Square-Cordial Graph.

Proof:

Let $V(K_{1,n}) = \{[u, u_i: 1 \le i \le n]\}$ and

$E(K_{1,n}) = \{[(uu_i): 1 \le i \le n]\}$.

Define $f: V(K_{1,n}) \rightarrow \{0,1\}$.

The vertex labeling are,

$f(u) = 1$

$$f(u_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n$$

The induced edge labelling are,

$$f^*[(uu_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n$$

When $n = 2m + 1,\ m > 0$

$v_f(0) = v_f(1) = \frac{n+1}{2}$ and

$e_f(0) = \frac{n-1}{2}$

$e_f(1) = \frac{n+1}{2}$.

Therefore, Star $K_{1,n}(odd)$satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence,Star $K_{1,n}(odd)$is Analytic Mean Square-Cordial Graph.

For example, the Analytic Mean Square-Cordial Labelling of$K_{1,5}$is shown in figure 3.2.

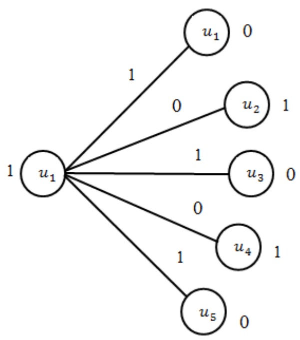

Figure 3.2:$K_{1,5}$

Theorem: 3.3

Star $K_{1,n}(even)$is Analytic Mean Square-Cordial Graph.

Proof:

Let $V(K_{1,n}) = \{[u, u_i: 1 \leq i \leq n]\}$ and

$E(K_{1,n}) = \{[(uu_i): 1 \leq i \leq n]\}$.

Define $f: V(K_{1,n}) \rightarrow \{0,1\}$.

The vertex labeling are,

$f(u) = 1$

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labelling are,

$$f^*[(uu_i)] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

When $n = 2m + 2, \ m \geq 0$

$v_f(0) = \frac{n}{2}$.

$v_f(1) = \frac{n}{2} + 1$ and

$e_f(0) = e_f(1) = \frac{n}{2}$.

Therefore, Star $K_{1,n}(even)$satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence,Star $K_{1,n}(even)$is Analytic Mean Square-Cordial Graph.

For example, the Analytic Mean Square-Cordial Labeling of$K_{1,6}$is shown in figure 3.4.

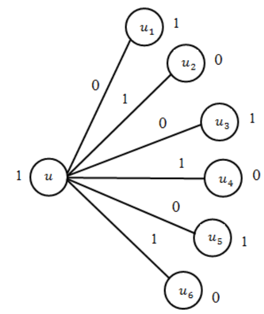

Figure 3.4:$K_{1,6}$

Theorem: 3.5

Subdivided Star $< K_{1,n}: n >$is Analytic Mean Square-Cordial Graph.

Proof:

Let $V(< K_{1,n}: n >) = \{[u, u_i, v_i: 1 \leq i \leq n]\}$ and

$E(< K_{1,n}: n >) = \{[(uu_i) \cup (u_i v_i): 1 \leq i \leq n]\}$.

Define $f: V(< K_{1,n}: n >) \rightarrow \{0,1\}$.

The vertex labeling are,

$f(u) = 1$

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labelling are,

$$f^*[(uu_i)] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f^*[(u_i v_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

When $n = 2m + 1, \ m > 0$

$v_f(0) = n$

$v_f(1) = n + 1$ and

$e_f(0) = e_f(1) = n$.

When $n = 2m + 2, \ m \geq 0$ $v_f(0) = n$

$v_f(1) = n + 1$ and

$e_f(0) = e_f(1) = n$.

Therefore, Subdivided Star $< K_{1,n}: n >$satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

For example, the Analytic Mean Square-Cordial Labeling of$< K_{1,5}: 5 >$is shown in figure 3.6.

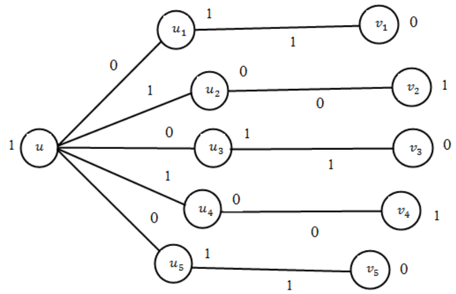

Figure 3.6:$< K_{1,5}: 5 >$

Theorem: 3.7

Tree $Tr(n)$is Analytic Mean Square-Cordial Graph.

Proof:

Let $V(Tr(n))$= $\{[u_i, v_{i1}, v_{i2}, w_{i1}, w_{i2}: 1 \leq i \leq n]\}$ and

$E(Tr(n))$=$\{[(u_i u_{i+1}): 1 \leq i \leq n - 1] \cup [(u_i v_{i1}) \cup (v_{i1} w_{i1}): 1 \leq i \leq n] \cup$

$[(w_{i1} w_{i2}) \cup (v_{i1} v_{i2}): 1 \leq i \leq n]\}$

Define $f: V(Tr(n)) \rightarrow \{0,1\}$.

The vertex labeling are,

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f(v_{ij}) = \begin{cases} 0 & i \equiv 0{,}1 \bmod 2 \\ 1 & i \equiv 0{,}1 \bmod 2 \end{cases} \quad 1 \leq i \leq n, \ j = 1,2$$

$$f(w_{ij}) = \begin{cases} 0 & i \equiv 0{,}1 \bmod 2 \\ 1 & i \equiv 0{,}1 \bmod 2 \end{cases} \quad 1 \leq i \leq n, \ j = 1,2$$

The induced edge labelling are,

$$f^*[(u_i u_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f^*[(v_{i1} v_{i2})] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f^*[(u_i v_{i1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f^*[(v_{i1} w_{i1})] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

When $n = 2m - 1, \ m > 0$

$v_f(0)$ $=5m - 3, \ m > 0$

$v_f(1)$ $=5m - 2, \ m > 0$and

$e_f(0)$ $=e_f(1)$ $=5m - 3, \ m > 0$

When $n = 2m, \ m > 0$

$v_f(0) = v_f(1) = 5m, \ m > 0$and

$e_f(0) = 5m - 1, \ m > 0$

$e_f(1) = 5m, \ m > 0$

Therefore, Tree $Tr(n)$satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, Tree $Tr(n)$is Analytic Mean Square-Cordial Graph.

For example, the Analytic Mean Square-Cordial Labeling of $Tr(6)$is shown in figure 3.2.

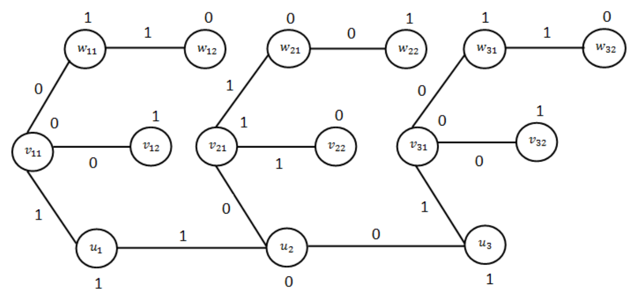

Figure 3.6:$Tr(6)$

REFERENCES

[1] Gallian J.A, A Dynamic Survey of Graph Labelling, The Electronic Journal of Combinatorics, 6 (2001) #DS6

[2] Harray. F, Graph Theory, Adadison-Wesley Publishing Company Inc, USA, 1969

[3] A.Nellai Murugan and V.Baby Suganya, A study on cordial labeling of Splitting Graphs of star Attached C_3 and $(2k+1)C_3$ ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 142 -147. I.F 6.531

[4] A.Nellai Murugan and V.Brinda Devi , A study on Star Related Divisor cordial Graphs ,ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 169-172. I.F 6.531

[5] A.Nellai Murugan and M. Taj Nisha, A study on Divisor Cordial Labeling Star Attached Path Related Graphs, ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 173-178. I.F 6.531

[6] Nellai Murugan and V .Sripratha, Mean Square Cordial Labelling , International Journal of Innovative Research & Studies, ISSN 2319-9725 ,Volume 3, Issue 10Number 2 ,October 2014, PP 262-277.

[7] A.Nellai Murugan and A.L Poornima ,Meanness of Special Class Of Graphs, International Journal of Mathematical Archive. ISSN 2229-5046, Vol 5 , issue 8, 2014, PP 151-158.

[8] A.Nellai Murugan and A.Mathubala, Path Related Homo- cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348-7968,Vol.2, Issue 8 ,August. 2015,PP 888-892. IF 1.50,IBI-Factor 2.33

[9] A.Nellai Murugan ,V.Selva Vidhya and M Mariasingam, Results On Hetro- cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348-7968,Vol.2, Issue 8 ,August. 2015,PP 954-959. IF 1.50,IBI-Factor 2.33

[10] A.Nellai Murugan , and R.Megala, Path Related Relaxed Cordial Graphs , International Journal of Scientific Engineering and Applied

[11] Science (IJSEAS) - ISSN: 2395-3470,Volume-1, Issue-6, September ,2015 ,PP 241-246 IF ISRA 0.217.

[12] A.Nellai Murugan and J.Shiny Priyanka, Tree Related Extended Mean Cordial Graphs, International Journal of Research -Granthaalayah, ISSN 2350-0530,Vol.3, Issue 9 ,September. 2015, PP 143-148. I .F . 2.035(I2OR).

[13] A.Nellai Murugan and S.Heerajohn, Cycle Related Mean Square Cordial Graphs, International Journal Of Research & Development Organization – Journal of Mathematics,Vol.2, Issue 9 ,September. 2015, PP 1-11.

[14] A.Nellai Murugan and A.Mathubala, Special Class Of Homo- Cordial Graphs, International Journal Emerging Technologies in Engineering Research, ISSN 2524-6410,Vol.2, Issue 3 ,October 2015, PP 1-5.

[15] A.Nellai Murugan , and R.Megala ,Tree Related Relaxed Cordial Graphs, International Journal of Multi disciplinary Research & Development , ISSN: 2349-4182,Volume-2, Issue-10, October ,2015 ,PP 80-84 . IF 5.742.

[16] A.Nellai Murugan , and A.Mathubala ,Cycle Related Homo-Cordial Graphs , International Journal of Multi disciplinary Research & Development, ISSN: 2349-4182,Volume-2, Issue-10, October ,2015 ,PP 84-88 . IF 5.742.

[17] L. Pandiselvi , A. Nellai Murugan , and S. Navaneethakrishnan , Some Results on Near Mean Cordial , Global Journal of Mathematics , ISSN 2395-4760,Volume 4,No : 2,August-2015, PP 420-427.

[18] A.Nellai Murugan and V.Selvavidhya , Path Related Hetro- Cordial Graphs, International Journal Emerging Technologies in Engineering Research, ISSN 2524-6410,Vol.2, Issue 3 ,October 2015, PP 9-14.

[19] A.Nellai Murugan and S.Heerajohn, Special Class of Mean Square Cordial Graphs, International Journal Of Applied Research ,ISSN 2394-7500,Vol.1, Issue 11 ,Part B , October 2015, PP 128-131.I F 5.2

[20] A.Nellai Murugan and J.Shinny Priyanka , Extended Mean Cordial Graphs of Snakes ,International Journal Emerging Technologies in Engineering Research, ISSN 2524-6410,Vol.3, Issue 1 ,October 2015, PP 6-10.

[21] A.Nellai Murugan and R.Megala Special Class of Relaxed Cordial Graphs ,International Journal Emerging Technologies in Engineering Research, ISSN 2524-6410,Vol.3, Issue 1 ,October 2015, PP 11-14.

[22] A.Nellai Murugan and P. Iyadurai Selvaraj, Cycle and *Armed Cap Cordial graphs*, Global Scholastic Research Journal of Multidisciplinary , ISSN 2349-9397,Volume , Issue 11, October 2015, PP 1-14. ISRA 0.416

[23] A.Nellai Murugan and J.Shinny Priyanka , Path Related Extended Mean Cordial Graphs ,International Journal of Resent Advances in Multi- Disciplinary Research, ISSN 2350-0743,Vol.2, Issue 10 ,October 2015, PP 0836-0840. I F 1.005.

[24] A.Nellai Murugan and G.Devakiruba., Divisor cordial labeling of Book and $_{Cn}$ @ $_{K1,n.,}$ OUTREACH, A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 86-92.

[25] A. Nellai Murugan and P. Iyadurai Selvaraj, Path Related Cap Cordial Graphs, OUTREACH, A Multi- Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 100-106.

[26] Nellai Murugan, and A.Meenakshi Sundari, Product Cordial Graph of Umbrella and C_4 @ $S_{n.,}$ OUTREACH , A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 113 – 119.

[27] Nellai Murugan, and V.Sripratha, Mean Square Cordial Labeling, OUTREACH, A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 125 – 131.

[28] A.Nellai Murugan and A.L.Poornima, Meanness of Special Class of Graph, OUTREACH, A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 140 – 145.

[29] A.Nellai Murugan and G.Esther, Mean Cordial Labeling of Star, Bi-Star and Wheel, OUTREACH , A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 155 – 160.

[30] A. Nellai Murugan and P. Iyadurai Selvaraj, Cycle & Armed Cap Cordial Graphs, International Journal of Mathematical Combinatorics, ISSN 1937 1055 , Volume II, June 2016, Pp. 144-152.

Authors

Dr. **A. Nellai Murugan**, Head and Associate Professor in the Department of Mathematics, V.O.Chidambaram College, Tuticorin which is affiliated to Manonmaniam Sundaranar University, Tirunelveli. His area of interest is Operations Research and Graph Theory. At present five scholars are doing research under his guidance in the field of Graph Theory. In his credit, he has published more than 100 research articles in the reputed national and international journals.

Miss. **N. Roselin,** is a second year Post Graduate student in Mathematics. Her field in the project is Graph Theory and She published One article in the International Journal. This article is original contribution in her project work.